THE 4 DAY GED MATH REVIEW

QUICK & EASY NOTES TO PREPARE FOR THE GED MATH SECTION
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Basic Algebra Terms and Concepts

**Number Operations**

- operators in math +, -, x, ÷
- \( x + y \) "the sum of \( x \) and \( y \)"
- \( x - y \) "the difference of \( x \) and \( y \)"
- \( xy \) "the product of \( x \) and \( y \)"
- \( x ÷ y \) or \( \frac{x}{y} \) "the quotient of \( x \) and \( y \)"

**Variables**

- letters or symbols that represent a number in algebra

Examples \( x, y, z \) or \( π \)

This symbol is called "\( π \)" it's approximately 3.14
Order of Operations

• Very important!
• The correct order you work out a math problem
• Remember by “PEMDAS”

Please Excuse My Dear Aunt Sally…..

What is the order? PEMDAS

1. P - parenthesis - do what’s inside first
2. E - exponents / powers next
3. M/D - multiplication / division from left to right
4. A/S - addition / subtraction from left to right

Example: \[3(6+2^2) ÷ 15 + 5\]

P: \[3(6+2^2) ÷ 15 + 5\]
E: \[3(6+4) ÷ 15 + 5\]
M: \[3(10) ÷ 15 + 5\]
D: \[30 ÷ 15 + 5\]
A: \[2 + 5\]

7 Answer
Basic Algebra Terms and Concepts

- Translating Verbal and Algebraic Phrases
  - Know key phrases, examples
    - “a number” - variable
    - “is” - = sign
    - 4x + y
      - “four times a number plus another number”
    - “the product of two numbers plus 3”
      - 4x + 3

- Definition of Equations, Inequalities and Solutions

  **Equations** - math statements that have a equal sign, =.
  Examples: x + 2 = 8

  Equations with a variable are called “open sentences”

  **Inequalities** - math statements that have a <, >, ≤, ≥, ≠ symbol
  Examples, 9 > 2 or 3x + 6 ≤ 12

  **Solutions** - Any value for a variable that makes an equation or inequality true
Converting Fractions/Decimals

Convert a fraction to a decimal
Divide numerator by denominator
Example, \( \frac{1}{4} = 1 \div 4 = .25 \) (use calculator)

Convert a decimal to a fraction
Write as fraction - Example .3 = \( \frac{3}{10} \)

"Three-tenths"

LCM/LCD

The LCM is the lowest number that two or more numbers divide into.

Example, the LCD of 3 and 4 is 12

Why? 12 is the lowest number 3 and 4 divide into

The LCD is the LCM of two or more denominators

Find the LCD/LCM by prime factoring
Example, Lcm 30, 40

\[
\begin{align*}
3 & \quad 2 & \quad 5 & \quad 2 & \quad 2 & \quad 5 \\
\text{LCM} &= 2 \cdot 2 \cdot 2 \cdot 5 \\
\text{LCM} &= 120
\end{align*}
\]
Multiplying and Dividing Fractions

Multiplying Fractions

\[
\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15} = \frac{4}{9}
\]

- Change any mixed-numbers into improper fractions

Example,
\[
3\frac{1}{2} \cdot \frac{1}{5} = \frac{7}{2} \cdot \frac{1}{5} = \frac{7}{10}
\]

Dividing Fractions

- Need to make division problems into multiplication

- Change any mixed-numbers into improper fractions

- Flip the second fraction (divisor) - then multiply fractions

Example,
\[
\frac{3}{4} \div \frac{1}{5} = \frac{3}{4} \cdot \frac{5}{1} = \frac{15}{4}
\]
Fractions and Decimals

Adding and Subtracting Fractions

- Need to have common denominators; this requires finding the LCM of the denominators
- Once the denominators are common—add or subtract the numerators
- Change any mixed-numbers into improper fractions

Example: $3\frac{1}{4} + 3\frac{3}{5}$

\[
\begin{align*}
\text{Find LCD, } & \quad \text{LCM of 4, 5 is 20} \\
\text{LCM} = 20
\end{align*}
\]

\[
\begin{align*}
\text{Adjust} & \quad \frac{13}{4} + \frac{3}{5} \\
\text{Numerator} & \quad \frac{5 \cdot 13}{5} + \frac{3 \cdot 4}{4}
\end{align*}
\]

\[
\begin{align*}
\text{denominators are now common add/subtract numerators} & \quad \frac{65}{20} + \frac{12}{20} \\
\text{Answer} & \quad \frac{65 + 12}{20}
\end{align*}
\]
Real Numbers

Real Number System

- numbers on the "Real" number line
  - whole number
  - natural number
  - integers
  - Rational
  - Irrational

- Real numbers can express everyday concepts
  - $10^{\text{th}}$ ten dollars of debt
  - $87.5\text{ ft.}$ 87.5 feet above ground

Adding and Subtracting Real Numbers

- $-6 + -2$ same sign, add keep sign $= -8$
  - $10 - 13$ turn subtraction into "plus-negative"
  - $10 + -13$ subtract, keep sign of the number with greatest absolute value $= -3$

Negative of a negative = positive
- $(-7) + -5 = 7 + -5 = 2$

Subtract, keep sign of the number with greatest absolute value
Real Numbers

- **Multiplying and Dividing Real Numbers**

  \[ 2 \cdot -5 = -10 \text{ \quad + \quad times \quad - = negative} \]
  \[ -3 \cdot -10 = 30 \text{ \quad - \quad times \quad - = positive} \]
  \[ 12 \div -4 = -3 \text{ \quad + \quad divided by \quad - = negative} \]
  \[ \frac{-40}{-10} = 4 \text{ \quad - \quad divided by \quad - = positive} \]

- **Distributive Property**

  \[ 2(3 + 1) = 2(3) + 2(1) = 6 + 2 = 8 \]
  \[ 6(x - 2) = 6(x) + 6(-2) = 6x - 12 \]
  \[ -3(4x - 1) = -3(4x) + -3(-1) = -12x + 3 \]

- **Combining Like Terms**

  \[ \frac{4}{2}x + 2x = 6x \]
  Like terms = same variable - same power
  (can add)

  \[ 4x^2 + 2x = 4x^2 + 2x \]

  NOT Like terms = same variable - different powers!
  (can not add)

  \[ 2(x + 3) + 4(x - 1) = 2x + 6 + 4x - 4 \]
  Distribute first, add any like terms
  \[ = 2x + 4x + 6 + -4 \]
  \[ = 6x + 2 \]
Equations

**One-Step Equations**

\[
\begin{align*}
\quad x - 6 &= -14 \\
+6 & \quad +6 \\
\hline
\quad x &= -8 \\
\end{align*}
\]

add 6 to both sides to get x by itself.

\[
\begin{align*}
\quad -3y &= 30 \\
-3 & \quad -3 \\
\quad y &= -10
\end{align*}
\]

here divide both sides by -3 to isolate y.

*watch your integers!*

\[
\begin{align*}
\quad m + 7 &= -2 \\
-7 & \quad -7 \\
\hline
\quad m &= -9
\end{align*}
\]

subtract 7 from both sides to isolate m.

**Two-Step Equations**

\[
\begin{align*}
\quad \frac{2}{5}m + 3 &= 2 \\
-3 & \quad -3 \\
\hline
\quad \frac{2}{5}m &= -1 \\
\quad \frac{5}{2} \cdot \frac{2}{5}m &= \frac{5}{2} \cdot -1 \\
\quad m &= -\frac{5}{2}
\end{align*}
\]

Isolate variable term first use + or - in this case subtract 3 from both sides.

Then \( \times \) or \( \div \) to solve the remaining one-step equation.
Multi-Step Equations

Steps to follow
1. Distribute, combine like terms
2. All variable terms on left side
3. Numbers on right
4. Solve the basic one-step equation

\[-3(x-6) + 4(x+1) = 7x-10\]
\[-3x + 18 + 4x + 4 = 7x - 10\]
\[x + 22 = 7x - 10\]
\[7x - 7x = -22\]
\[-6x = -32\]
\[-6x = -32\]
\[x = \frac{-32}{-6} = \frac{16}{3}\]
Formulas and Literal Equations

\[ P = 2l + 2w \]

OK, only think of "l" as a variable

\[ P = 2l + 2w \]

think of these variables as numbers

So, solve for l as any other equation, for example \( 10 = 2l + 2(3) \)

\[ P = 2l + 2w \]

\[ 2l + 2w = P \]

\[ -2w -2w \]

\[ 2l = P - 2w \]

\[ \frac{2l}{2} = \frac{P - 2w}{2} \]

\[ l = \frac{P - 2w}{2} \]

← solution
Inequalities

Graphing Inequalities

< or > use open circle
≤ or ≥ use closed circle

Examples

\[ x > 3 \], \[ x < -7 \], \[ x \geq 10 \], \[ x \leq -2 \]

Verifying Solutions to Inequalities

Replace the variable(s) with values - determine if the resulting statement is true or false.

True = value is solution
False = value is NOT solution

Example, Is \( x = -6 \) a solution to \( 2x - 1 \leq 9 \)?

\[ 2(-6) - 1 \leq 9 \]
\[ -12 - 1 \leq 9 \]
\[ -13 \leq 9 \] True, \( x = -6 \) is a solution
Inequalities

**Solving an Inequality and Graphing the Solution**

- Use same steps as if you were solving an equation
- If you divide both sides of the inequality by a negative number - reverse the inequality sign
- Graph the simplified inequality

**Example**

Solve and graph the solution 

\[-2(4x + 1) < 10\]

\[-2(4x + 1) < 10\]
\[-8x - 2 < 10\]
\[+2 +2\]

\[-8x < 12\]
\[\frac{-8}{-8} \frac{-12}{-8}\]

\[x > -\frac{12}{8}\]

(inequality sign reversed - divided both sides by negative number)

Graph \(x > -\frac{3}{2}\)

\[\begin{array}{c}
\cdot \quad -\frac{3}{2} \\
\cdot \quad 0
\end{array}\]
Solve and Graph a Two-Variable Inequality

Steps
1. Graph line
2. Draw a solid line for $\geq$ or $\leq$ inequalities
3. Draw a dashed line for $>$ or $<$ inequalities
4. Test a $(x,y)$ point - shade true region

Example, solve and graph $2x + 6y \leq -18$

Test $(0,0)$
$2x + 6y \leq -18$
$2(0) + 6(0) \leq -18$
$0 \leq -18$ False - shade the region below the line
Graphing Linear Equations

Graphing Lines with One Variable

\[ x = 5 \quad \text{x = number, vertical line} \]
\[ y = -3 \quad \text{y = number, horizontal line} \]
Slope of a Line

Defined as $m \text{(slope)} = \frac{\text{rise}}{\text{run}}$

Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$(1, 6), (2, 3)$ ← these points are on the line

$m = \frac{6 - 3}{1 - 2} = \frac{3}{-1} = -3$

slope = -3
Graphing Linear Equations

Graphing Lines with Two Variables

Three methods
1. Construct a table
2. \( y = mx + b \)  slope-intercept method
3. \( x \) and \( y \) Intercept Method

Graphing a line using a table
1. construct a table by plugging in a \( x \)-value into the equation
2. Next solve for \( y \)
3. The \( x \)-value and \( y \) solution are a point on the line \((x, y)\)

Example, graph \( y + 2x = 6 \)

\[ y + 2x = 6 \]

plug-in 1, then solve for \( y \)

\[ y + 2(1) = 6 \]
\[ y + 2 = 6 \]
\[ y = 4 \]

\[ \begin{array}{c|c}
\hline
x & y \\
\hline
1 & 4 \\
5 & \ \\
\hline
\end{array} \]

the point \((1, 4)\) is on the line, try another \( x \)-value, \( x = 5 \)
Graphing Linear Equations

\[ y + 2x = 6 \]
\[ y + 2(5) = 6 \]
\[ y = -4 \]

The point \( (5, -4) \) is also on the line.

Plot the two points and draw the line.

The graph of the line is \( y = 2x + 6 \).
Graphing Linear Equations

Graphing lines using $y = mx + b$

Next use the slope to find a second point on the line.

First plot $b$, this is the $y$-intercept (one point on line).

Graph $y = \frac{2}{3}x + 2$

Point A

Use slope to get point B

Line $y = \frac{2}{3}x + 2$
Graphing Linear Equations

Graphing lines using the XY-Intercepts

1. Plug in $x=0$ and solve for $y$ - this pair of points is the $y$-intercept, $(0, y)$.
2. Plug in $y=0$ and solve for $x$ - this pair of points is the $x$-intercept, $(x, 0)$.
3. Plot the points - draw the graph

Graph $2x + 3y = -12$

<table>
<thead>
<tr>
<th>$x$-intercept</th>
<th>$y$-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x + 3(0) = -12$</td>
<td>$2(0) + 3y = -12$</td>
</tr>
<tr>
<td>$2x = -12$</td>
<td>$3y = -12$</td>
</tr>
<tr>
<td>$x = -6$</td>
<td>$y = -4$</td>
</tr>
</tbody>
</table>

$(0, -4)$

Graph of line $2x + 3y = -12$

$(0, -4)$
Writing the Equation of a Linear Equation

- **Using Slope-Intercept Form** \((y=mx+b)\)

  Use formula \(y=mx+b\), use given information to find \(m, b\).

  Example - Find the equation of a line that has a slope of 2 and passes through the point \((3,9)\).

  \[y = mx + b\]

  \[q \rightarrow m \uparrow \uparrow \text{solve for } b\]

  \[9 = 2(3) + b\]
  \[9 = 6 + b\]
  \[b = 3\]  
  \[\text{equation of line } y=2x+3\]

- **Using Point-Slope Form**

  \[y - y_1 = m(x - x_1) \rightarrow \text{Point-Slope Formula}\]

  \[\uparrow \uparrow \uparrow \uparrow \]

  plug in these values - then solve for \(y\)

  write equation as \(y=mx+b\)

  Example - Find the equation of a line that has a slope of 2 and passes through the point \((3,9)\).

  \[y - y_1 = m(x - x_1)\]

  \[\uparrow \uparrow \uparrow \uparrow \]

  \[q \rightarrow 2 \rightarrow 3\]

  \[y - 9 = 2(x - 3)\]

  \[y - 9 = 2x - 6\]

  \[y + 9 = 2x + 9\]

  \[y = 2x + 3 \leftarrow \text{equation of line}\]
Write the Equation of a Line Given Two Points

- Can use point-slope formula or \( y = mx + b \)
- Both methods require you to find the slope, \( m \)

Example - Find the equation of a line that passes through (1,2) and (3,-8)

\[
m = \frac{2 - (-8)}{1 - 3} = \frac{2 + 8}{-2} = \frac{10}{-2} = -5
\]

\( m = -5 \)

\[y = mx + b\]

use \( m = -5 \) and one point on the line, (1,2) or (3,-8)

\( m = -5 \) point (1,2)

\[
y = m x + b
\]

\[2 = -5(1) + b\]

\[2 = -5 + b\]

\[b = 7\]

Equation of line, \( y = -5x + 7\)

Point-slope formula, use \( m = -5 \) and one point on the line, (1,2) or (3,-8)

\( m = -5 \) point (3,-8)

\[
y - y_1 = m(x - x_1)
\]

\[
\uparrow \quad \uparrow \quad \uparrow
\]

\[-8 \quad -5 \quad 3\]

\[
y - (-8) = -5(x - 3)
\]

\[
y + 8 = -5x + 15
\]

\[-8 \quad -8\]

Equation of line \( \Rightarrow y = -5x + 7 \)
Writing the Equation of a Linear Equation

Standard Form

Write equation in $Ax + By = C$ form

Example,
Write $y = -\frac{1}{2}x + 5$ in Standard form

$y = -\frac{1}{2}x + 5$
$+\frac{1}{2}x + \frac{1}{2}x$

$y + \frac{1}{2}x = 5$

Best Fitting Lines

Objective - writing an equation of a line that follows the trend of points on the graph

1. Draw a line that “best” fits trend
2. Pick two points on line - find equation

Use (7,9) and (-4,-2) to find $y = mx + b$
Solve By Graphing

- Graph the lines in the system.
- The point where the lines intersect is the solution.
  \[(2, 4)\]

Example,

Solve the system by graphing:

\[
\begin{align*}
  y - x &= 2 \\
  2y + x &= 10
\end{align*}
\]
Systems

Substitution Method

Steps
1. Solve for one variable in one equation
2. Substitute into the other equation—making one equation with one variable
3. Solve the one variable equation
4. Use the solution to solve for other variable

Example,

\[
\begin{align*}
\begin{cases}
y - x &= 2 \\ 2y + x &= 10
\end{cases}
\end{align*}
\]

Step 1

\[
y = x + 2 \quad \text{step 2}
\]

\[
\begin{align*}
2y + x &= 10 \\
2(x + 2) + x &= 10 \\
x + 4 + x &= 10 \\
x + 4 &= 10
\end{align*}
\]

Step 3

\[
\begin{align*}
x &= 2 \\
y &= x + 2 \\
y &= 2 + 2 = 4
\end{align*}
\]

the solution is \((2, 4)\)
Linear Combination

Steps
1. Line up respective variables in column
2. If needed multiply one or both equations by a number(s) to create two terms that are opposite
3. Add the equations to eliminate a term
4. Solve the one variable equation
5. Use the solution to solve for the other variable

\[ \begin{align*}
    y - x &= 2 & \text{step 1} \\
    2y + x &= 10 \\
    \hline
    y - x &= 2 & \text{step 2 - the x terms are opposite, they will cancel when the equations are added} \\
    2y + x &= 10 \\
    \hline
    3y &= 12 \\
    y &= 4 \\
    \hline
    \text{solution (2, 4)}
\end{align*} \]
System of Linear Inequalities

Steps
1. Graph each linear inequality
2. Find the intersection of the lines, solve the system — use any method
3. The overlapping region is the solution

Example — Solve and graph

\[
\begin{align*}
    y - x &< 2 \\
    2y + x &\geq 10
\end{align*}
\] (dash line)

the solution to this system (2,4)
**Linear Programming**

What is it? A mathematical method to optimize (find the maximum or minimum) of a linear system

General Steps
1. Use given information to write the equations of the linear system - these equations are called the constraints
2. Use the given information to write the objective function - this is the equation we want to "optimize."
3. Graph the constraint equations
4. Use your knowledge of systems to find the ordered pairs (point) where each pair of constraints intersect.
5. Each point of intersection on the graph is called a vertex - one of the vertices is the ordered pair that optimizes the objective function; test each to find the max or min value.
Absolute Value

Absolute Value Definition

What is absolute value? It's the distance a number is from zero.

\[ -7 \text{ away from 0} \]

\[ 4 \text{ from 0} \]

Absolute value is always positive because distance is always measured in positive units.

Evaluating Absolute Value Expressions

\[ |-3| \]

the bars around the -3 is the absolute value symbol

So, for example \[ |-3| \] means "how far is -3 from zero?"

answer: 3 units

\[ -3 \text{ units } \]

\[ 0 \text{ units} \]

\[ 3 \text{ units} \]

\[ |-3| = 3 \]
Absolute Value

Graphing Absolute Value Equations

Basic shape of an absolute value graph is a “V”

Steps

1. Find the vertex of the graph
2. Determine if the graph is V-shaped or upside down V, i.e. ∨-shaped.
3. Graph equation, plot more points to make a more accurate graph

Example, graph \( y = |x-5| + 7 \)

Vertex, \((x, y)\) point of the “V”

Step 1

\[ y = |x-5| + 7 \]

Find by setting to zero, solve for \(x\)

\[ x - 5 = 0 \]
\[ x = 5 \]

Vertex located at \((5, 7)\)

Now, find \((x, y)\)

\[ y = |x-5| + 7 \]
Absolute Value

Step 2

\[ y = |x - 5| + 7 \]

- sign is positive, upward V graph
- when sign is negative upside-down V,

Step 3

Graph \( y = |x - 5| + 7 \)
- vertex \((5, 7)\), upward-V graph
**Absolute Value**

**Solving Absolute Value Equations**

Steps

1. Isolate the absolute value part of the equation
2. Set-up two equations
3. Solve both equations - absolute value equations always have two solutions

Example,

Solve the absolute value equation

\[ 2|x + 9| - 4 = 12 \]

\[ \begin{align*} 
    2|x + 9| - 4 &= 12 \\
    \frac{2}{2}|x + 9| &= \frac{16}{2} \\
    |x + 9| &= 8 \\
\end{align*} \]

**Step 1**

Isolate \( |x + 9| \)

\[ 1 \cdot |x + 9| = 8 \quad \text{two equations} \]

\[ x + 9 = 8 \quad x + 9 = -8 \]

**Step 2**

\[ \begin{align*} 
    x + 9 &= 8 \\
    -9 -9 &= -9 -9 \\
\end{align*} \]

\[ x = -1 \quad \text{solutions} \rightarrow x = -17 \]
Absolute Value

Solving Absolute Value Inequalities

Two cases - less than and greater than

Steps
1. Isolate the absolute value part of the inequality
2. Set up compound inequality
3. Solve compound inequality - solutions
4. Graph solution

Example

\[ 3|2x + 2| < 9 \]

Step 1
\[ \frac{3|2x + 2|}{3} < \frac{9}{3} \]
\[ |2x + 2| < 3 \]

-3 ≤ 2x + 2 ≤ 3

Step 2
\[ \begin{align*}
|2x + 2| &< 3 \\
\text{take out of abs. val.} &
\end{align*} \]

-3 ≤ 2x + 2 ≤ 3

Surround by ±3

Step 3
\[ \begin{align*}
-3 &≤ 2x + 2 ≤ 3 \\
-3 - 2 &≤ 2x ≤ 3 - 2 \\
-5 &≤ 2x ≤ 1
\end{align*} \]

Sign

Step 4
\[ \begin{align*}
\frac{-5}{2} &≤ x ≤ \frac{1}{2}
\end{align*} \]

Solutions

Graph solution

-5/2

1/2
Properties of Exponents

\[
a^m \cdot a^n = a^{m+n} \quad \frac{2^3 \cdot 4^2}{2^7} = \frac{2^5}{2^7} = 2^{-2} = 2^4 \quad \frac{x^5}{x^2} = x^{5-2} = x^3
\]

\[
(a^m)^n = a^{mn} \quad (2^3)^5 = 2^{3\cdot5} = 2^{15} \quad (x^6)^3 = x^{6\cdot3} = x^{18}
\]

\[
(a^m b^n)^q = a^{mq} b^{nq} \quad (2^3 3^4)^5 = 2^3 \cdot 3^4 \cdot 5 = 2^3 \cdot 3^4 \cdot 5 = 2^4 \cdot 3^4 = x^8 \cdot y^{10}
\]

\[
\frac{a^m}{a^n} = a^{m-n} \quad \frac{2^6}{2^2} = 2^{6-2} = 2^4 \quad \frac{x^9}{x^5} = x^{9-5} = x^4
\]

\[
a^{-n} = \frac{1}{a^n} \quad 2^{-7} = \frac{1}{2^7} \quad x^{-2} = \frac{1}{x^2}
\]

\[
a^0 = 1 \quad 3^0 = 1 \quad y^0 = 1
\]

Use properties to simplify expressions

Example

\[
\frac{(x^2)^3 \cdot y^6 \cdot x^4}{y^2} = \frac{x^6 \cdot x^4 \cdot y^6}{y^2} = x^{6+4} \cdot y^6 = x^{10} \cdot y^4
\]
Powers and Exponents

Scientific Notation

What is it? A way to rewrite very large or small numbers using powers to the base of 10.

Example: 2,960,000 → scientific notation → 2.96 × 10^6

Write a number in scientific notation:

4,670,000 → 4.67 × 10^5

First use digits to write a number between 1 and 10. The exponent is the number of digits between decimal point.

Note - decimal point moved left, power of 10 positive; moved right power of 10 negative.

Write 0.00019 in scientific notation:

0.00019 → 1.9 × 10^{-4}

Decimal moved 4 right.
Powers and Exponents

**Compound Interest**

What is it? Growth rate that increases over time; used in investing money - your money will make you more money over time, this is called “interest”

Formula \[ A = P(1 + r)^t \]

- \( A \): future value
- \( P \): principal
- \( r \): interest rate (as decimal)
- \( t \): time in years

**Example**

$5000 is invested for 10 years at 6%
compounded every year. How much is the investment at the end of the 10 years?

\[ A = P(1 + r)^t \]
\[ P = 5000 \text{ starting amount} \]
\[ r = 0.06, \text{ 6% as decimal} \]
\[ t = 10 \text{ years} \]
\[ A = 5000(1.06)^{10} \]
\[ A = 5000(1.79) \]
\[ A = 8954.23 \]

**Answer - $8954.23**

**Exponential Growth and Decay**

\[ y = a^x \]

- **Growth** (\( a > 1 \))
- **Decay** (\( 0 < a < 1 \))
Adding and Subtracting Polynomials

- Add like terms
- Write answer in standard form
- Be careful when subtracting

Example

\[ \begin{align*}
2x^2 + 3x + 1 \\
+ \quad x^3 + 6x^2 - 5 \\
\hline
x^3 + (2x^2 + 6x^2) + 3x + 1 - 5
\end{align*} \]

Answer in standard form = \[ x^3 + 8x^2 + 3x + -4 \]

Example

\[(5x^2 - 2x + 4) - (x^2 - 7x + 2)\]

First distribute negative sign to

\[ 5x^2 - 2x + 4 + -x^2 + 7x - 2 \]

Add like terms = \[ 4x^2 + 5x + 2 \]

Write in standard form
Polynomials and Factoring

Multiplying Polynomials

Methods
1. F.O.I.L. (Binomials Only)
2. Distribute
3. Special Rules

F.O.I.L. (First, Outer, Inner, Last)
Example \((2x+1)(x+5)\)

\[
\begin{align*}
F & \rightarrow O \\
(2x+1)(x+5) & = 2x(x) + 2x(5) + 1(x) + 1(5) \\
I & \rightarrow L \\
& = 2x^2 + 10x + 1x + 5 \\
\text{Answer} & = 2x^2 + 11x + 5
\end{align*}
\]

Distribute Method
Example \((x-5)(3x^2+2x-4)\)

\[
\begin{align*}
(x-5)(3x^2+2x-4) & = 3x^3 + 2x^2 - 4x \\
(x-5)(3x^2+2x-4) & = -15x^2 - 10x + 20 \\
\text{Answer} & = 3x^3 - 13x^2 - 14x + 20
\end{align*}
\]
Special Polynomial Multiplication Rules

- **3 cases**

  **Case 1** \((a+b)(a-b) = a^2 - b^2\)
  
  Example \((2x + 3)(2x - 3) = (2x)^2 - (3)^2\)
  
  \[\text{answer} = 4x^2 - 9\]

  **Case 2** \((a+b)^2 = a^2 + 2ab + b^2\)
  
  Example \((x+7)^2 = (x)^2 + 2(x)(7) + (7)^2\)
  
  \[\text{answer} = x^2 + 14x + 49\]

  **Case 3** \((a-b)^2 = a^2 - 2ab + b^2\)
  
  Example \((3x-2)^2 = (3x)^2 - 2(3x)(2) + (2)^2\)
  
  \[\text{answer} = 9x^2 - 12x + 4\]

Factoring the Greatest Common Factor

- Always **first step in factoring**
- Reverse of distributive property

Example, Factor the **gcf**

\[
\begin{align*}
4x + 10 & \quad \rightarrow \quad 2(2x + 5) \\
6x^2 + 3x & \quad \rightarrow \quad 3x(2x + 1) \\
x^3y^2 - xy & \quad \rightarrow \quad xy(x^2y - 1)
\end{align*}
\]

**Note:** need to know properties of exponents
Factoring Quadratic Trinomials

2 cases

Case 1 - leading term is $1x^2$

Example: $x^2-4x-5$

Factors: $(x-5)(x+1)$

Plug in -5 and 1 in this case the factors are -5 and 1

 verrrrrrrrrrrrrrrrrr

Case 2 - leading term is not $1x^2$

Example: $2x^2-7x-4$

Factors: $(2x)(x)$

Factors must add up to middle term

$(2x+1)(x-4)$

$1x-8x=-7x$

Middle term

$2x^2-7x-4$
Polynomials and Factoring

Special Factoring Rules

Use the special product rules to find factors

1. 3 cases

**Case 1** \((a+b)(a-b) = a^2 - b^2\)

\[(2x + 3)(2x - 3) = (2x)^2 - (3)^2 = 4x^2 - 9\]

Example factor 4\(x^2 - 9\)

Factors = \((2x + 3)(2x - 3)\)

**Case 2** \((a + b)^2 = a^2 + 2ab + b^2\)

Example factor \(x^2 + 14x + 49\)

\[(x + 7)^2 = (x)^2 + 2(x)(7) + (7)^2 = x^2 + 14x + 49\]

Factors

**Case 3** \((a - b)^2 = a^2 - 2ab + b^2\)

Example factor 9\(x^2 - 12x + 4\)

\[(3x - 2)^2 = (3x)^2 - 2(3x)(2) + (2)^2 = 9x^2 - 12x + 4\]

Factors
Quadratic Equations

Solve by Taking Square Roots

- Always two solutions in quadratic equations
- Can not take the square-root of a negative number - in the Real Number system

Example,
\[
\sqrt{9} \quad \sqrt{-9} \quad \text{Not a Real Number -}
\]
\[
+3 \quad -3 \quad \text{the answer is a Complex number}
\]

Steps
1. Isolate the "x^2" term
2. Take the square-root of both sides

Example
\[
2x^2 - 4 = 10
\]
\[
\begin{array}{c}
+4 +4 \\
\end{array}
\]
Step 1
\[
\frac{2x^2 = 14}{2} \quad \frac{x^2 = 7}{2}
\]
Step 2
\[
\sqrt{x^2} = \sqrt{7} \quad \text{two solutions}
\]
\[
x = \pm \sqrt{7}
\]
Quadratic Equations

Graphing Quadratic Equations

Steps
1. Find the vertex ("U"-shape)
2. Graph - the shape is always a parabola

Example \( y = 2x^2 + 4x - 8 \)

Finding vertex - write equation in standard form \((\text{highest} \rightarrow \text{lowest power}, \ ax^2 + bx + c = 0\))

\[
y = 2x^2 + 4x - 8
\]

Vertex is located at \( \left( \frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \)

\[
a = 2 \quad b = 4 \quad c = -8
\]

Find \( \frac{-b}{2a} = \frac{-4}{2(2)} = \frac{-4}{4} = -1 \)

Find \( f\left(\frac{-b}{2a}\right) \) - plug in \( \frac{-b}{2a} \) into equation

\[
y = 2x^2 + 4x - 8
\]

\[
= 2(-1)^2 + 4(-1) - 8 = 2(1) + -4 - 8 = -10
\]

Vertex \((-1, -10)\), \( y = 2x^2 + 4x - 8 \)

\( x^2 \) term is positive - graph upward parabola

\( x^2 \) term negative - downward

Graph upward parabola from \((-1, -10)\)

\[ y = 2x^2 + 4x - 8 \]
Quadratic Equations

**Quadratic Formula**

Very important! Need to master it to solve quadratic equations

When you have $ax^2 + bx + c = 0$

the solutions are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example Solve

$x^2 - 9x = -8$

First write in standard form

$x^2 - 9x + 8 = 0$

$a = 1, b = -9, c = 8$

Plug in values into formula

$x = \frac{9 \pm \sqrt{81 - 32}}{2}$

$x = \frac{9 \pm \sqrt{49}}{2}$

Solutions $x = \frac{10}{2}, x = \frac{2}{2}$
Quadratic Equations

Solve Quadratic Equations by Factoring

- Need to know how to factor polynomials
- Only works when you can factor - otherwise use quadratic formula
- Based on zero-product property

Example: \( x^2 - 9x + 8 = 0 \)
\[
(x - 1)(x - 8) = 0
\]

One of these terms must be zero, because that's the only way you can make the equation (left side) equal to zero (right side)

Solve by setting both factors equal to zero

\[
x - 1 = 0 \quad \Rightarrow \quad x = 1
\]

\[
x - 8 = 0 \quad \Rightarrow \quad x = 8
\]

Solutions

The Discriminant - Type of Roots

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\( b^2 - 4ac \) part is the Discriminant

When:

\[
b^2 - 4ac = 0 \quad \Rightarrow \quad 1 \text{ double root} \\
\]

\[
b^2 - 4ac > 0 \quad \Rightarrow \quad 2 \text{ real roots} \\
\]

\[
b^2 - 4ac < 0 \quad \Rightarrow \quad \text{No real roots}
\]

Note: “root” - means solutions
Completing the Square

A method to rewrite a quadratic equation in such a way as to solve by taking the square-root of both sides.

Example:

1. Write in Standard form $(ax^2 + bx + c = 0)$
   
   $(x^2 - 9x + 8 = 0)$

2. Move $c$ to the right side
   
   $x^2 - 9x = -8$

3. Add $(\frac{b}{2})^2$ to both sides
   
   $x^2 - 9x + (\frac{9}{2})^2 = -8 + (\frac{9}{2})^2$

4. Factor
   
   $\left(x - \frac{9}{2}\right)^2 = \frac{49}{4}$

5. Solve by taking $\sqrt{}$ both sides
   
   $x - \frac{9}{2} = \pm \frac{7}{2}$

6. Solutions
   
   $x = \frac{7}{2} + \frac{9}{2} = \frac{16}{2} = 8$
   
   $x = -\frac{7}{2} + \frac{9}{2} = \frac{2}{2} = 1$
Quadratic Equations

Graphing Quadratic Inequalities

Graph using same steps as linear inequalities

\[ y < 2x^2 + 4x - 8 \]

Border is \( \cdots \cdots \)

Test a point, I like \( (0,0) \)

\[ y < 2(0)^2 + 4(0) - 8 \]
\[ 0 < 2(0)^2 + 4(0) - 8 \]
\[ 0 < -8 \]

\( \uparrow \)

False statement—shade under parabola
Introduction to Functions and Relations

- Relations are a collection of ordered pairs, \((x, y)\) points.
- Functions are a special type of relation, one that pairs one \(x\) value with only one \(y\) value.

Functions and relations have various forms to include:

- **Graphs**
- **Sets**
- **Tables**
- **Equations** \((y = f(x))\)
- **Function notation** \(f(x) = 2x + 1\)

Note:
We call the \(x\)-numbers the **Domain** and \(y\)-numbers the **Range**.
Functions and Relations

Introduction to Functions and Relations

Test a relation to determine if it is a function -

- Can use the VLT - Vertical Line Test
- Or use a mapping diagram

Example using VLT

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

passes each x goes to only one y (Function)

Example using mapping

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Fails 1 is paired with 4 and 6; (Not a function)
**Function Operations**

Given functions you can +, -, x, ÷ them.

Examples,

\[ f(x) = 3x \quad g(x) = x + 2 \]

\[ f(x) + g(x) = 3x + x + 2 = 4x + 2 \]

\[ f(x) - g(x) = 3x - (x + 2) = 2x - 2 \]

\[ f(x) \cdot g(x) = 3x(x + 2) = 3x^2 + 6x \]

\[ \frac{f(x)}{g(x)} = \frac{3x}{x + 2} \]

Evaluating a function—plug in value and simplify.

Example, \( f(x) = 2x^2 \)

Say "f of 3" \( f(3) = 2(3)^2 = 2 \cdot 9 = 18 \)

**Inverse Functions**

Function and its Inverse

Domains and Range are switched.
Find an inverse function

Steps
1. replace the $f(x)$ with $y$
2. switch the $x$ and $y$ in equation
3. solve for $y$

Example

$f(x) = 2x + 4$

Step 1 $\rightarrow$ $y = 2x + 4$

Step 2 $\rightarrow$ $x = 2y + 4$

Step 3
\[
\begin{cases}
2y + 4 = x \\
2y = x - 4 \\
y = \frac{x - 4}{2}
\end{cases}
\]

Inverse notation $\rightarrow f^{-1}(x) = \frac{x - 4}{2}$

$\begin{align*}
f(f^{-1}(x)) &= f^{-1}(f(x)) \\
&= x
\end{align*}$

The composite of these functions are equal to $x$

$\begin{align*}
f(f^{-1}(x)) &= 2\left(\frac{x - 4}{2}\right) + 4 \\
&= x - 4 + 4 \\
&= x
\end{align*}$

Showing this is how you verify an inverse function
Functions and Relations

- **Graphing Functions**
  - Algebra students already have been graphing many functions - to include,
    - lines
    - Absolute Value
    - Quadratic

- **Linear and Non-Linear Functions**
  - Linear functions are lines, \( y = mx + b \)
  - Some non-linear functions are
    - Quadratic: \( y = a(x-h)^2 + k \)
    - EXPONENTIAL: \( y = a^x \)
    - Absolute Value: \( y = |x-h| + k \)

- **Composite Functions**
  When you evaluate a function with another function
  Example, \( f(x) = 2x + 1 \)  \( g(x) = 3x \)
  \( f(g(x)) = 2(3x) + 1 \)
  \( = 6x + 1 \)
Functions and Relations

- Special Functions
  - Piecewise Functions
  - Recursive
  - Greatest Integer

Rational Expressions and Equations

- Ratios and Proportions

  Ratio - comparing two numbers by division, the numbers count the same concept / usually same unit of measure
  Example \( \frac{1 \text{ Teacher}}{20 \text{ Students}} \)

  Rate - comparing two numbers by division, the numbers count different concepts / usually different unit of measure

  Proportion - two equal ratios or rates
  - The cross-products are equal
    Example \( \frac{1}{2} \times \frac{3}{6} \) cross-product
      
      \[
      1 \times 6 = 2 \times 3 \\
      6 = 6
      \]
  - Use the cross-product to solve various rate and ratio problems
Rational Expressions and Equations

Percent

- Percent - is a ratio that compares a number to 100.

Example, \( \frac{70}{100} \) is 70 percent = 70%

Find percent of a number

- Change percent to decimal, multiply by number

Example 60% of 90

\[ \downarrow \]

Divide by 100 to change to decimal

\( \frac{.60}{90} \times 90 = 54 \)

Other percent problems

- Use an equation or proportion to solve

Example, 15 is what percent of 75?

Equation method -

\[ x \cdot \frac{75}{75} = 15 \]

Answer

\[ \frac{.2 \times 100}{100} = 20\% \]

Change decimal to % multiply by 100

Percent - proportion method

-looking for % \[ \frac{x}{100} = \frac{15}{75} \]

- Use cross-product to solve

\[ 75x = 100(15) \]

\[ 75x = 1500 \]

\[ x = 20\% \]
Rational Expressions and Equations

Direct and Inverse Variation

\[ y = kx \quad \text{Direct} \]
\[ yx = k \quad \text{Inverse} \]
\[ k = \text{constant of variation} \]

Steps to solve variation problems
1. Identify if the relationship between the variables is direct or inverse
2. Use the general variation formula and information in problem to solve for \( k \)
3. Write a specific equation with the \( k \)-value
4. Use the equation from step 3 to solve the problem

Example, \( x \) and \( y \) vary directly, when \( x = 3 \)
\( y = 6 \). What is \( y \) when \( x = 10 \)?

Step 1. \( y = kx \)
Step 2. \( 6 = k \cdot 3 \)
Solve for \( k = 2 \)

Step 3. \( y = 2x \)
Step 4. \( y = 2(10) \)
\[ y = 20 \quad \text{Answer} \]
Rational Expressions and Equations

**Simplifying Rational Expressions**
- Cancel out like factors
- Know how to factor

\[
\frac{10x^2y}{15x^5y^2} = \frac{2 \cdot 5 \cdot x \cdot x \cdot x \cdot y}{3 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y} = \frac{2}{3x^3y}
\]

Factor
\[
\frac{2x + 8}{(x - 4)(x + 4)}
\]

\[
= \frac{2(x + 4)}{(x - 4)(x + 4)}
\]

\[
= \frac{2}{x - 4}
\]

**Multiplying and Dividing Rational Expressions**
- Follow same rules as numeric fractions
- Know how to multiply polynomials
- Always simplify your results

Examples,

\[
\frac{3x}{x + 1} \cdot \frac{2x + 4}{5} = \frac{3x(2x + 4)}{5(x + 1)} = \frac{6x^2 + 12x}{5x + 5}
\]

\[
\frac{3x}{x + 1} \div \frac{2x + 4}{5} = \frac{3x}{(x + 1)(2x + 4)} \cdot \frac{5}{x + 1}
\]

Flip, make problem into multiplication

\[
\frac{15x}{(x + 1)(2x + 4)}
\]
Rational Expressions and Equations

Adding and Subtracting Rational Expressions

- Follow the same rules as adding / subtracting fractions
- Know how to find the LCD of a rational expression
- Simplify your results

Examples,

\[
\frac{7x}{5x-3} - \frac{x}{5x-3} = \frac{7x-x}{5x-3} = \frac{6x}{5x-3}
\]

\[
\text{Same denominators, subtract numerators}
\]

\[
\frac{2x}{(x-1)(x+3)} + \frac{5}{(x+3)}
\]

\[
\text{Not common denominators, to fix multiply (x-1) to the numerator/denominator of}
\]

\[
\frac{5}{(x+3)} \cdot \frac{(x-1)}{(x-1)} = \frac{5(x-1)}{(x+3)(x-1)}
\]

\[
\frac{2x}{(x-1)(x+3)} + \frac{5(x-1)}{(x+3)(x-1)} \quad \text{Now we can add}
\]

\[
= \frac{2x + 5(x-1)}{(x-1)(x+3)} = \frac{7x - 5}{(x-1)(x+3)} \quad \text{Answer}
\]
Solving Rational Equations

Steps
1. Clear fractions by multiplying both sides of the equation by the LCD
2. Solve the remaining equations
3. Check your solutions—ignore “extraneous” solutions (any solution that creates a false statement when you check it)

* When you have one single-fraction rational expression equal to another you may use the cross-product to solve

Examples \[ \frac{x+1}{4} = \frac{3}{6} \]

use cross-product \[ 6(x+1) = 12 \]

\[ 6x + 6 = 12 \]

\[ 6x = 6 \]

\[ x = 1 \]

\[ \frac{x+1}{4} = \frac{3}{6} \]

Check \[ x = 1 \]

\[ \frac{2}{4} = \frac{3}{6} = \frac{1}{2} \] True, \( x = 1 \) is the solution

\[ \frac{x}{x+2} + \frac{1}{3} = 4 \]

Clear fractions – \( \text{LCD} = 3(x+2) \)
\[ 3(x+2) \left( \frac{x}{x+2} + \frac{1}{3} = 4 \right) \]

\[ 3(x+2) \cdot \frac{x}{x+2} + 3(x+2) \cdot \frac{1}{3} = 3(x+2) \cdot 4 \]

\[ 3x + x + 2 = 12(x+2) \]

\[ 4x + 2 = 12x + 24 \]

Check \( x = -\frac{11}{4} \)

in original equation

\[ -8x = 22 \]

\[ x = -\frac{22}{8} = -\frac{11}{4} \]

\[ -\frac{11}{4} \rightarrow \frac{x}{x+2} + \frac{1}{3} = 4 \]

\[ \frac{-\frac{11}{4}}{-\frac{11}{4} + 2} + \frac{1}{3} = 4 \]

\[ \frac{-\frac{11}{4}}{-\frac{3}{4}} + \frac{1}{3} = 4 \]

\[ \frac{-11}{4} \div \frac{-3}{4} = \rightarrow \frac{11}{3} + \frac{1}{3} = 4 \]

\[ \frac{12}{3} = 4 = 4 \]

**True, left = right**

\( x = -\frac{11}{4} \) is the solution
Radical Expressions and Equations

Simplifying Radicals

• Look for perfect square factors

\[
\text{Perfect squares:} \quad 4, 9, 16, 25, 36, \ldots \quad \sqrt{4} \quad \sqrt{9} \quad \sqrt{16} \quad \sqrt{25} \quad \sqrt{36} \\
2 \quad 3 \quad 4 \quad 5 \quad 6 \quad \ldots
\]

Example

\[
\sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}
\]

↑ Factor ↑ separate radicals

Operations with Radicals

• Adding and Subtracting - very similar to adding and subtracting like terms

Examples

\[
3\sqrt{7} + 2\sqrt{7} = 5\sqrt{7}
\]

↑ ↑
SAME

Note-
always simplify radical terms before adding and subtracting

\[
3\sqrt{7} + 2\sqrt{6}
\]

↑ ↑
Not the same - can't add

• Multiplying/Dividing radicals - follow these rules

\[
\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}
\]

Example

\[
\sqrt{2} \cdot \sqrt{3} = \sqrt{6}
\]

\[
\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}
\]

Example

\[
\frac{\sqrt{16}}{\sqrt{23}} = \frac{\sqrt{16}}{\sqrt{23}} = \frac{4}{23}
\]

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Solving Radical Equations

Steps
1. isolate radical
2. raise power to both sides- clear radical
3. Solve and check solution- disregard extraneous solutions (these solutions produce false statements when you verify)

Example \(2\sqrt{x+1} - 3 = 15\)

Step 1 \[2\sqrt{x+1} - 3 = 15 \quad +3 +3 \]
\[2\sqrt{x+1} = 18 \quad \frac{2}{2} \]

Step 2 \[(\sqrt{x+1})^2 = 9^2 \]
\[x + 1 = 81 \quad 2 \cdot 9 - 3 = 15 \]
\[x = 80 \quad 18 - 3 = 15 \quad 15 = 15 \quad \text{True} \]

Therefore \(x = 80\) is the solution
Distance and Midpoint Formulas

- Distance formula finds the distance between two points
- Mid-point formula finds the point that is halfway between two points

Distance Formula
\[ d = \sqrt{(x-x_1)^2 + (y-y_1)^2} \]

Example, find the distance between 
(2,4) and (5,10)

\[
d = \sqrt{(2-5)^2 + (4-10)^2} \\
= \sqrt{(-3)^2 + (-6)^2} \\
= \sqrt{9 + 36} \\
= \sqrt{45} \approx 6.70
\]
Radical Expressions and Equations

Distance and Midpoint Formulas

\[
\text{Mid-point formula } \left( \frac{x + x_1}{2}, \frac{y + y_1}{2} \right)
\]

Example, find the mid-point between \((6,2)\) and \((2,10)\)

\[
\begin{align*}
\left( \frac{x + x_1}{2}, \frac{y + y_1}{2} \right) &= \left( \frac{6 + 2}{2}, \frac{2 + 10}{2} \right) \\
&= \left( \frac{8}{2}, \frac{12}{2} \right) \\
&= (4,6)
\end{align*}
\]

\((4,6)\) is halfway between \((6,2)\), \((2,10)\)

MP = \((4,6)\)

Radical Expressions and Equations

The Pythagorean Theorem

- Finds the length of the legs of a right triangle
- If you know two of the lengths of a right triangle you can find the third

The Pythagorean Theorem

Given right triangle

\[
\text{Note - } c \text{ is always the longest leg}
\]

\[
a^2 + b^2 = c^2
\]
Radical Expressions and Equations

The Pythagorean Theorem

Examples,

Find the length of the missing legs

\[ a^2 + b^2 = c^2 \]

\[ 4^2 + 3^2 = c^2 \]

\[ 16 + 9 = c^2 \]

\[ 25 = c^2 \]

Find \( c \) by taking square root of both sides

\[ c^2 = 25 \]

\[ \sqrt{c^2} = \sqrt{25} \]

\[ c = 5 \quad \text{answer} \]

\[ \begin{align*}
10 \\
8
\end{align*} \]

plug-in

\[ a^2 + b^2 = c^2 \]

\[ 8^2 + x^2 = 10^2 \]

\[ 64 + x^2 = 100 \]

\[ -64 \quad -64 \]

\[ x^2 = 36 \]

\[ \sqrt{x^2} = \sqrt{36} \]

\[ x = 6 \quad \text{answer} \]
Area of Basic Figures

**Triangle** - \[ A = \frac{1}{2}bh \]

- \( b = 8 \) \( h = 2 \)
- \( A = \frac{1}{2}(8)(2) \)
- \( A = 8 \text{ units squared} \)

**Parallelogram** - \( A = bh \)

- \( b = 10 \) \( h = 2 \)
- \( A = (10)(2) \)
- \( A = 20 \text{ units squared} \)

**Trapezoid** - \( A = \frac{1}{2}h(b_1 + b_2) \)

- \( h = 3 \)
- \( b_1 = 5 \)
- \( b_2 = 9 \)
- \( A = \frac{1}{2}(3)(5 + 9) \)
- \( = \frac{1}{2}(3)(14) \)
- \( = (3)(7) \)
- \( A = 21 \text{ units squared} \)
**Circles - Area and Circumference**

\[
\text{Area} = \pi r^2 \quad \text{Circumference} = D\pi = 2\pi r
\]

\(\pi \approx 3.14\)

( use calculator value for more accurate answer)

**Circumference Example**

\[C = D\pi\]
\[C = 6\pi\]

\[C \approx 6(3.14) \approx 18.84 \text{ units}\]

**Area Example**

\[A = \pi r^2\]
\[A = \pi (3)^2\]

\[A = 9\pi\]

\[A \approx 9(3.14)\]

\[A \approx 28.26 \text{ units squared}\]
Surface Area and Volume of Basic Figures

**Surface Area of Basic Figures**

Cylinder - $SA = 2\pi rh + 2B$

Example $r = 2$ $h = 8$ $B =$ area of circle

$SA = 2\pi (2)(8) + 2\pi (2)^2$

$= 32\pi + 8\pi$

$= 40\pi$ or $40(3.14) \approx 125.6 \text{ ft}^2$

Prism - $SA = ph + 2B$

Example $p =$ perimeter of base

$p = 2 + 5 + 4 = 11''$

$B = \frac{1}{2}(2)(4) = 4 \text{ in}^2$

$SA = (11)(7) + 2(4)$

$SA = 77 + 8 = 85 \text{ in}^2$

Pyramid - $SA = n\left(\frac{1}{2}bh\right) + B$

Example $n =$ # sides = 4

$l =$ length of side = 6

$b =$ length of base = 4

$B =$ area of base

$B = (4)(4) = 16$

$SA = 4\left(\frac{1}{2}\cdot4\cdot6\right) + 16$

$SA = 48 + 16 = 64 \text{ units squared}$
Surface Area and Volume of Basic Figures

\[
\text{Cone} \quad SA = \pi rl + B
\]

Example
\[
\begin{align*}
  r &= 2 \\
  l &= 7
\end{align*}
\]

\[
B = \pi (2)^2 = 4\pi
\]
\[
SA = \pi (2)(7) + 4\pi
\]
\[
SA = 14\pi + 4\pi = 18\pi \text{ units squared}
\]

\[
\text{Sphere} \quad SA = 4\pi r^2
\]

Example
\[
\begin{align*}
  r &= 3
\end{align*}
\]

\[
SA = 4\pi (3)^2 = 4\pi \cdot 9 = 36\pi \text{ or } 36(3.14) \approx 113.04 \text{ units squared}
\]
Volume of Basic Figures

**Cube**

\[ V = s^3 \]

Example

\[ V = (3)^3 = 27 \text{ in}^3 \]

**Cylinder**

\[ V = Bh \]

Example

\[ B = \pi(2)^2 = 4\pi \]
\[ h = 8 \]
\[ V = (4\pi)(8) = 32\pi \approx 100.53 \text{ ft}^3 \]

**Prism**

\[ V = Bh \]

Example

\[ B = \frac{1}{2}(2)(4) \]
\[ h = 7 \]
\[ V = (4)(7) = 28 \text{ units cubed} \]

**Pyramid**

\[ V = \frac{1}{3}Bh \]

Example

\[ B = (4)(4) = 16 \]
\[ h = 5 \]
\[ V = \frac{1}{3}(16)(5) = \frac{80}{3} = 26.67 \text{ units cubed} \]
Volume of Basic Figures

Cone: \[ V = \frac{1}{3} Bh \]

Example:
\[ B = \pi r^2 = 4\pi \]
\[ h = 8 \]
\[ V = \frac{1}{3} (4\pi)(8) \]
\[ V = \frac{32\pi}{3} \approx 33.49 \text{ units cubed} \]

Sphere: \[ V = \frac{4}{3} \pi r^3 \]

Example:
\[ r = 3 \]
\[ V = \frac{4}{3} \pi (3)^3 \]
\[ = \frac{4}{3} \pi (27) \]
\[ = 4\pi(9) = 36\pi \approx 113.04 \text{ units cubed} \]